Key Curriculum Press Response to Indiana Review of Discovering Geometry

SUMMARY

Overall rating:	Important Mathematical Ideas:	Skills and Procedures:	Mathematical Relationships:
Strong (3-4)	Strong (3-4)	Moderate (2-3)	Strong (3-4)

Standards for Mathematical Practice: Standards 2, 3, 7, 8 3 Standards 1, 4, 5, 6 4

While we acknowledge the very positive rating of *Discovering Geometry*, we want to respond to some comments that get to the heart of our materials:

Proof not a focus. Weakness in summarizing ideas/proofs

In the review of *Discovering Geometry*, proof was seen as "not a focus" or a "weakness in summarizing ideas/proof." The development of proof skills in *Discovering Geometry*, while different from most textbooks, was deliberate and was based on educational research that recommends delaying formal proof until after students have more concrete experience with geometric figures and their properties. A supplemental titled *Tracing Proof in Discovering Geometry* is provided for each teacher in the teacher resources. This supplemental helps teachers understand the research behind the purposeful design of *Discovering Geometry*. Included below is a summary of the guide to developing geometric thinking used in *Discovering Geometry*.

The most widely recognized model for the development of geometric thinking was created by Dina van Hiele-Geldof and Pierre van Hiele (1973). The van Hiele model describes five levels that students' progress through as they develop readiness for proving theorems within a deductive system. The model has been supported through independent experimentation and also through practical experience—many geometry teachers find it describes accurately the different levels of thinking that they observe in their own students. Over the years, limitations in the van Hiele model—for example, that it focuses on how students deal with geometric properties—have been addressed by later researchers. Other aspects of mathematical thinking, such as different roles of proof, are important in the geometry curriculum, so the levels of geometric thinking that guide *Discovering Geometry* extend the van Hiele scheme to include work by others, including Presmeg (1991), Orton (1995), and de Villiers (various).

The van Hieles found that most students begin high school geometry at Level 0 (Visual) or Level 1 (Analytic), yet most high school geometry courses are taught in a way that assumes students begin at Level 3 (Formal Deduction and Proof). The investigations in *Discovering Geometry* begin at Levels 0 and 1 and provide progressive experiences designed to help students move toward Level 3. All along, the text also offers challenges to students who have attained levels higher than the prerequisite level.

The statement found in the Common Core State Standards on proof is:

> **Prove geometric theorems.** *(Encourage multiple ways of writing proofs, such as in narrative **paragraphs**, using **flow diagrams,** in **two-column format**, and using **diagrams** without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning. Implementation of G.CO.10 may be extended to include concurrence of perpendicular bisectors and angle bisectors as preparation for G.C.3 in Unit 5.*

Key Curriculum Press Response to Indiana Review of Discovering Geometry

In Discovering Geometry, Chapter 2 is an example of where students first progress from inductive to deductive reasoning through the use of paragraph proof, defined on page 201 as a deductive argument that uses written sentences to support its claim to reasons. Examples can be found on page 204–205, Exercises 14, 15, 18, and 19. In Lesson 4.6— Exercises 1–9 ask students to prove statements they say are true. (So— first answer — is it true? Then prove it!) Students move to a more systematic form of proof in Chapter 4 where flowchart proofs are introduced, defined to be a logical argument presented in the form of a flowchart. In Lesson 4.7, basically every exercise is a proof. See Lesson 4.8, where again, flowchart proofs and paragraph proofs are assigned (Exercises 4–10). And Chapter 5 (Discovering and Proving Polygon Properties)—there are a few proofs in each lesson, with Lesson 5.7 completely focused on proof, explicitly scaffolding students from filling in flowchart proofs to writing them from scratch. A focus on creating flowchart proofs continues in Chapters 6 – 12. In Chapter 13 the two column proof format is introduced as the purpose of proof moves from justification to systemization. Conjectures from earlier in the text are formally "proven" and become theorems.
Students don't always do it alone.
Discovering Geometry engages students by giving them an opportunity to investigate mathematics in cooperative groups. As they work, group members will learn to plan together, brainstorm, determine and organize tasks, and communicate their individual and collective results. Students work together to make sense of the mathematics as they investigate, make conjectures, clarify or explain their thinking, discuss ideas, and look back on their findings. The investigations are significant pedagogical components. They are designed to engage students so that learning becomes important to them.